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Theodorsen's Propeller Performance with Rollup and Swirl in the Slipstream

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Introduction

THEODORSEN'S classic theory,[1] elegant and efficient as it is for the design of optimal propellers under specified conditions, suffers from an unrealistic high average static pressure in the slipstream. This high pressure has its impact on the expressions for predicted thrust, power, and efficiency. In earlier papers[2,3] this author has pointed out that the high pressure is inherent to the model with rigid vortex sheets in the slipstream. It is unrealistic to maintain the rigidity down to the Trefftz plane. The high pressure is eased by allowing rollup of the vortex sheets.

Theodorsen[1] models the propeller wake as a rigid backward moving (multiple) helical vortex sheet. Far downstream, in the region of the Trefftz plane, where the slipstream is supposed to be in equilibrium with its surroundings, the sheets move with the constant velocity w (in the following referred to in dimensionless form $\bar{w} = w/V$). A rigid vortex sheet, assumed to be force free, is not in equilibrium with its surroundings. Rollup ensures that the static pressure in the wake tends toward ambient pressure or lower. In the rolled-up model, the sheets start rolling up as soon as they leave the propeller blades. In the vicinity of the propeller the rollup has proceeded so little that the sheets are considered to have their original shape for the most part. The flow is still in development. For the velocities induced at the propeller, the development downstream is at first considered to be of minor importance. The design of the propeller is performed using the induced velocities at the propeller pertaining to the spiraling rigid sheets, uninfluenced by the rollup.

In Ref. 2 a rough implementation of the rollup has been presented to demonstrate its impact on the prediction of thrust and power coefficients and on efficiency. In that implementation, the average static pressure over the slipstream cross section was the ambient pressure. In a recent paper Ribner[4] directed attention to the fact that swirl reduces the interior pressure of the slipstream below ambient and that, therefore, the momentum equation (10) in Ref. 2 is a flawed equation. In the improved modification of the theory, presented in the following sections, using a more specified model of the slipstream incorporating rollup and swirl, the effects are taken into account in the momentum and energy balance and in the resulting expression for the efficiency.

In an example, referring to the eight-bladed propfan mentioned by Ribner in Ref. 4, the efficiency as predicted by the present theory is only 1.5% below the measured value of 82.3%. This is twice as close to the measured value as the prediction of the classical theory (see Ribner[4]). This result is interpreted as support of the present improved theory.

Modification of the Theory from Rigid to Rolled Up

Role of Edge Forces

When considering Theodorsen's[1] helicoidal slipstream at a distance from the propeller, it is worthwile to discuss the implicit consequence of the rigidity of the sheets. The singular edge forces on the rigid sheets, although everywhere normal to the propeller axis, play a fundamental role in keeping the slipstream together. In a sense they counterbalance the high pressure in the central region. In the two-dimensional model used as an example by Schouten,[3] it is clear at once that the effect of letting the sheets roll up is that the high pressure in the central region disappears. Abandoning the edge forces implies abandoning the high pressure in the central region.

In a three-dimensional model, things are complicated by the rotation of the propeller and the swirl in the slipstream. As long as only axial and radial velocities are involved, the simple two-dimensional reasoning applies. The complications come from the tangential velocity components v_θ in the slipstream. These components require a radial pressure gradient in the slipstream taking care of the curvature of the streamlines in the crossplane. In Theodorsen's[1] rigid-sheet model, the edge forces take care of this aspect as well as of balancing the high pressure inside the slipstream.

The edge forces are unrealistic (they do not exist in a free slipstream) and so is the high pressure in Theodorsen's model.

Low Static Pressure in the Slipstream with Swirl

We restrict the discussion to a model where far downstream all gradients in the axial direction are negligibly small. The swirl in the slipstream goes with a radial pressure gradient subject to the equilibrium condition

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = \varrho\frac{v_\theta^2(r)}{r}, \qquad p(r) = \int_0^{R_s} \varrho\frac{v_\theta^2(r)}{r}\,\mathrm{d}r \tag{1}$$

The distribution of swirl velocity $v_\theta(r)$ governs the static pressure in the slipstream. In combination with an (arbitrary) axial velocity distribution, the distribution of **total head** follows from

$$p_0(r) = p(r) + (\varrho/2)\left[v_z^2(r) + v_\theta^2(r)\right] \tag{2}$$

The condition at the edge of the slipstream is that the static pressure is the ambient pressure p_a. The positive outward pressure gradient (1) then implies that the static pressure in the slipstream has its maximum value p_a at the edge. Inside the slipstream, the pressure $p(r) \leq p_a$, with a minimum at the axis. Any tangential velocity distribution has its own radial pressure distribution. In constructing the equilibrium model we are free to allow any axial velocity distribution provided it goes with a distribution of total head as described by Eq. (2).

Development of Rigid Helicoid Sheet

We consider a slipstream model (see Fig. 1) that, at the propeller and close behind it, is Theodorsen's rigid helicoidal sheet model. The rollup of the sheets results far downstream in a simple averaged

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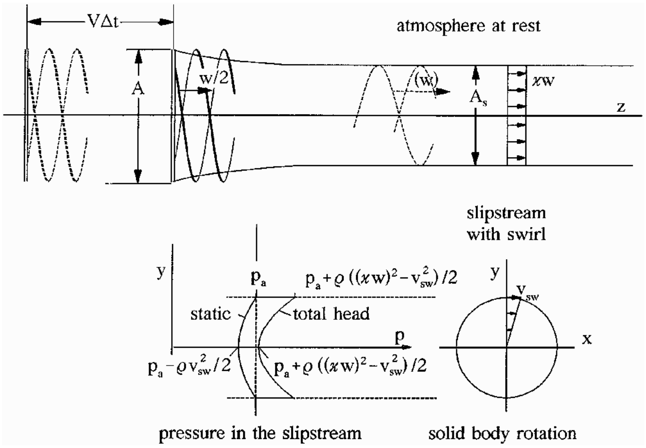

Fig. 1 Model of the rolled-up slipstream with swirl.

equilibrium model of the slipstream as a straight jet with constant axial velocity. The averaged axial vorticity in the central region models the slipstream rotating as a solid body. The tip vortices form a vortex sheet between the slipstream and the surrounding atmosphere. Static pressure and total head are decreasing towards the axis.

The model of the rigidly moving helical vortex sheets is described in Ref. 1; the model is assumed to be known to the reader. At the propeller blades the induced velocity is $w/2$, and the induced velocity averaged over the propeller area A is $\varkappa w/2$. These are the reference magnitudes for the rolled-up slipstream. The induced velocity on the rigid sheets in the region of the Trefftz plane is w, twice the induced velocity at the propeller blades. The average of the induced axial velocity in the rolled-up slipstream is $\varkappa w$, twice the average induced velocity at the propeller plane. Here $\varkappa$ is the mass coefficient, defined by Theodorsen,[1] together with the coefficients ε and μ as

$$\varkappa = \int \frac{v_z \,\mathrm{d}A_s}{w A_s}, \qquad \mu = \int \frac{v^2 \,\mathrm{d}A_s}{w^2 A_s}, \qquad \varepsilon = \int \frac{v_z^2 \,\mathrm{d}A_s}{w^2 A_s} \tag{3}$$

The rolled-up slipstream is modeled by a constant axial velocity $\varkappa w$ over a reduced area A_s. (Note that in this model the axial velocity distribution in the slipstream is assumed to develop in such a way that, for example, on the axis the zero induced axial velocity is brought up to the average value $\varkappa w$ in the Trefftz plane.) Continuity of mass flow then yields the relation between A_s and A,

$$(V + \varkappa w/2)A = (V + \varkappa w)A_s \tag{4}$$

In the following, to avoid confusion with $\varkappa w$, we refer to the axial-induced velocity as the constant velocity $v_z (= \varkappa w)$.

The slipstream is assumed to rotate as a solid body; the tangential velocities associated with this swirl are referred to as $v_{\mathrm{sw}}(r)$. The tangential velocity $v_{\mathrm{sw}}(R_s)$ at the boundary of the slipstream, referred to in the following as v_{sw}, is related to the axial velocity v_z in the slipstream by the spiraling tip vortices. They move downstream at the edge of the slipstream, with the average of the exterior and the interior velocity, that is, $v_z/2$. A propeller with B blades sheds B tip vortices, where the total circulation per revolution (n revolutions/s) of the propeller is distributed over a length $V + v_z/2$:

$$B\Gamma = \frac{(V + v_z/2)}{n} v_z \tag{5}$$

These same vortices represent the discontinuity in tangential velocity due to the solid-body rotation of the slipstream with cross section A_s and diameter $D_s = 2R_s$. Thus, for the tangential velocity at the edge determining ω_{sw}, we find

$$v_{\mathrm{sw}}(R_s) = B\Gamma/\pi D_s = v_z (V/\pi n D)(A/A_s)^{\frac{1}{2}} \times [1 + (v_z/2V)] = \omega_{\mathrm{sw}} R_s \tag{6}$$

The relation with the propeller is present in $v_z = \varkappa w$ and in the geometric advance ratio $J = V/\pi n D$. Inside the slipstream the tangential velocity is $v_{\mathrm{sw}} r = \omega_{\mathrm{sw}} r$. From Eq. (6) it follows that the swirl velocity ω_{sw} of the slipstream is proportional to the inverse of the angular velocity ω of the propeller:

$$\omega_{\mathrm{sw}} = (1/\omega)\varkappa\bar{w}(1 + \varkappa\bar{w}/2)\left(\pi V^2/A_s\right) \tag{7}$$

A rotating jet is held together by an outward pressure gradient that forces the projection of the streamlines on the crossplane to be circles. At the outer edge the static pressure is the ambient pressure p_a. Rotation as a solid body requires a pressure distribution in the slipstream to satisfy

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \varrho\omega_{\mathrm{sw}}^2 r, \qquad p(r) = p_a - \varrho\frac{v_{\mathrm{sw}}^2}{2}\left[1 - \left(\frac{r}{R_s}\right)^2\right]$$
$$\int_{A_s} [p(r) - p_a]\,\mathrm{d}A_s = -\frac{\varrho v_{\mathrm{sw}}^2}{4} A_s \tag{8}$$

Important Features of the Rolled-Up Sheet Model with swirl

The axial momentum flux in the slipstream of the rolled-up sheet model with swirl is equal to that of Theodorsen's model.[1]

The energy flux in the rolled-up sheet model with swirl is maintained equal to that of Theodorsen's model. The energy flux in the modified model then consists of structured flow energy, yielding total head, and losses due to unstructured flow energy or dissipated flow energy.

Thrust, Power, and Efficiency with the Rotating Slipstream

The momentum balance is

$$T_{\mathrm{air}} - \int_{A_s} [p(r) - p_a]\,\mathrm{d}A_s = \varrho v_z (V + v_z) A_s \tag{9}$$

Inserting the pressure distribution (7) then yields the thrust

$$T_{\mathrm{air}} = \varrho v_z (V + v_z) A_s - \varrho\frac{[v_{\mathrm{sw}}(R_s)]^2}{2}\frac{A_s}{2} \tag{10}$$

The thrust coefficient, following Theodorsen,[1] defined as $T/(\varrho V^2 A_s/2)$, then becomes

$$c_{s,S^*} = 2\varkappa\bar{w}(1 + \varkappa\bar{w}) - J^2(\varkappa\bar{w})^2(1 + \varkappa\bar{w}/2)(1 + \varkappa\bar{w})/2 \tag{11}$$

The total power required to deliver this thrust is found as the sum of the useful power TV and the kinetic energy left behind in the slipstream minus the work performed by the pressure forces on the surroundings:

$$P = TV + \frac{\varrho}{2}\int (V + v_z)\left\{v_z^2 + v_{\mathrm{sw}}^2(r) + v_{\mathrm{loss}}^2\right\}\mathrm{d}A_s - \int v_z[p_a - p_s(r)]\,\mathrm{d}A_s \tag{12}$$

The term $\varrho(v_{\mathrm{sw}}^2 + v_{\mathrm{loss}}^2)/2$ is interpreted to represent the kinetic energy that was present in Theodorsen's rigid sheets as $\varrho(v_\theta^2 + v_r^2)/2$. In the process of rolling up, the solid-body rotation is conserved as structured kinetic energy. The rest of the kinetic energy is present in the local field around the vortex kernels; it is administrated as lost kinetic energy in the term $\varrho v_{\mathrm{loss}}^2/2$. This interpretation then allows the quantification of the kinetic energy term in Eq. (12) as

$$\int (V + v_z)\left(v_z^2 + v_{\mathrm{sw}}^2 + v_{\mathrm{loss}}^2\right)\mathrm{d}A_s \approx \int (V + \varkappa w)v_{\mathrm{Theodorsen}}^2\,\mathrm{d}A_s \approx (V + \varkappa w)\varkappa w^2 A_s \tag{13}$$

The required power (12), thus, takes the form

$$P = -\varrho(V+\varkappa w)v_{\rm sw}^2 A_s/4 + \varrho\varkappa w(V+\varkappa w)(V+w/2)A_s \quad (14)$$

The power coefficient, defined as $c_P = P/(\varrho V^3 A_s/2)$, using Eq. (6) then becomes

$$c_{p,S^*} = 2\varkappa\bar{w}(1+\varkappa\bar{w})\{1+\bar{w}/2 - J^2\varkappa\bar{w}(1+\varkappa\bar{w}/2)(1+\varkappa\bar{w})/4\} \quad (15)$$

The efficiency of the propeller with the slipstream rotating as a solid body and an average axial velocity $\varkappa w$ is found to be

$$\eta_{S^*} = \frac{1 - J^2(1+\varkappa\bar{w}/2)\varkappa\bar{w}/4}{1+\bar{w}/2 - J^2(1+\varkappa\bar{w}/2)(1+\varkappa\bar{w})\varkappa\bar{w}/4} \quad (16)$$

As a check, we compare the efficiency of an eight-bladed propeller[5] based on expression (16) with the values mentioned by Ribner,[4] referring to that propeller (those values are based on rigid-sheet theory). For an accurate comparison we must retrieve the value of $\varkappa$ pertaining to the values mentioned by Ribner, $V/nD = 3.002$ and $w/V = 0.47$, yielding a value $(V+w)/(nD) = 4.413$. Theodorsen's[1] chart VI then yields, after interpolation, a mass coefficient $\varkappa = 0.12$. With these values, Eq. (16) yields an estimated efficiency $\eta = 80.8\%$. This value is twice as close to the measured value $\eta = 82.3\%$ as the estimated values mentioned by Ribner,[4] $\eta = 85.3$ and $\eta = 85.8\%$ obtained by Hanson,[5] and by Theodorsen's[1] rigid-sheet method.

Resulting Expressions for Thrust, Power, and Efficiency

The coefficients for thrust and power are nondimensionalized by $\varrho(V^2/2)A_s$, where A_s is the cross-sectional area of the slipstream:

$$c_{s,\rm Th} = 2\varkappa\bar{w}(1+\varepsilon/\varkappa\bar{w}) + 2\varkappa\bar{w}^2 - \mu\bar{w}^2$$

$$c_{s,\rm Sch} = 2\varkappa'\bar{w}(1+\varepsilon'/\varkappa'\bar{w}) + \varkappa'\bar{w}^2 - \mu'\bar{w}^2 \quad (17)$$

$$c_{s,S^*} = 2\varkappa\bar{w}(1+\varkappa\bar{w}) - J^2(\varkappa\bar{w})^2(1+\varkappa\bar{w}/2)(1+\varkappa\bar{w})/2$$

$$c_{p,\rm Th} = 2\varkappa\bar{w}(1+\varepsilon/\varkappa\bar{w}) + 2\varkappa\bar{w}^2 - \mu\bar{w}^2 + \mu\bar{w}^2 + 2\varepsilon\bar{w}^3$$

$$c_{p,\rm Sch} = 2\varkappa'\bar{w}(1+\varepsilon'/\varkappa'\bar{w}) + \varkappa\bar{w}^2 - \mu'\bar{w}^2 + \mu'\bar{w}^2 + \varepsilon'\bar{w}^3$$

$$c_{p,S^*} = 2\varkappa\bar{w}(1+\varkappa\bar{w})\{1+\bar{w}/2 - J^2\varkappa\bar{w}(1+\varkappa\bar{w}/2)(1+\varkappa\bar{w})/4\} \quad (18)$$

where subscripts Th refer to Theodorsen's rigid helicoid theory,[1] Sch to the results of Ref. 2, and S^* to the theory incorporating rollup and swirl as presented in this Note. The efficiencies associated with the expressions (17) and (18) are

$$\eta_{\rm Th} = \left[\frac{1}{(1+\bar{w}/2)}\right]\left[1+\left(\frac{\varkappa\bar{w}^3}{c_p}\right)\left(\frac{1}{2}-\varepsilon/\varkappa\right)\right]$$

$$\eta_{\rm Sch} = \frac{1}{(1+\bar{w}/2)} \quad (19)$$

$$\eta_{S^*} = \frac{1 - J^2\varkappa\bar{w}(1+\varkappa\bar{w}/2)/4}{1+\bar{w}/2 - J^2\varkappa\bar{w}(1+\varkappa\bar{w}/2)(1+\varkappa\bar{w})/4}$$

Discussion

Parameters

In principle, the parameters $\varkappa$, ε, and μ and $\varkappa'$, ε', and μ' are different in the rigid-sheet and rolled-up sheet model; therefore, Schouten's parameters are primed in Eqs. (17) and (18). Ribner[4] brought forward an objection to the implementation by Schouten[2] of the rollup in Theodorsen's[1] model. Ribner[4] states that use of the same values of the parameters in both models would lead to a false comparison of results (referred to by Ribner as "comparing apples to oranges"). Ribner is correct when he states that the models are different and that in principle the parameters must be carefully distinguished. Both models, however, are meant to estimate quantities in the slipstream that is generated by a propeller whose trailing vortex sheets immediately downstream of that propeller move as rigid helical sheets with the velocity $\bar{w}/2$, yielding an induced average velocity $\varkappa\bar{w}/2$. That is the basis of comparison of the models. Therefore, the average velocity $\varkappa\bar{w}/2$ at the propeller disk is the reference for induced velocities in both models; the parameters ε and μ no longer occur in the present modified model.

In the development of the slipstream from rigid to rolled-up, the mass flow through the propeller is not affected. In the crude model of Ref. 2 the effective modification consisted of the reduction of the average static pressure in the slipstream to ambient pressure. In a preceding section it has been pointed out that the static pressure over the cross section of the slipstream is governed by the boundary condition of ambient pressure and the radial distribution of swirl velocity. In the expressions for the thrust and power coefficients and for the efficiency, the average velocity $\varkappa\bar{w}$ and the geometrical advance ratio $J = V/\pi nD$ prove to be the governing parameters. The mass parameter $\varkappa$ as defined by Theodorsen[1] guarantees, in the product $\varkappa\bar{w}/2$, the relation between the propeller and the slipstream. When the swirl is neglected, the slipstream reduces to a straight jet with constant axial velocity (actuator model). Static pressure and total head are constant over the cross section. In the present amended model, the static pressure distribution in the slipstream is governed by the swirl. The structured velocities in the slipstream, v_z and $v_{\rm sw}$, are related to the extra total head, and this implicitly relates to the "pressure–velocity tradeoff," as referred to by Ribner.[4] Some of the kinetic energy is dissipated at constant pressure into random kinetic energy. All of the kinetic energy that dissipated or remained is taken care of in the power balance.

Actuator Disk Limit

No one will dispute the results of actuator disk theory. The actuator disk is to be considered as a propeller with an infinite number of blades rotating at an infinite angular velocity, $J = 0$, and $\varkappa = 1$. The model is simple; there is no rotation in the slipstream. The slipstream (having velocity $v_z = w$) is separated from the external field by a vortex sheet devoid of axial components. This vortex sheet moves at the mean of the velocities of the slipstream and the outer flow. The total head of the slipstream is raised by an amount $\Delta p_0 = \varrho w^2/2$ due to the unsteady pressure $-\varrho(\partial\Phi/\partial t)$ generated by the vortex sheets moving at the velocity $w/2$. The efficiency of the actuator surface is $\eta = 1/(1+\bar{w}/2)$, where $\bar{w} = w/V$. Here a flaw of Theodorsen's[1] theory is touched upon. Its actuator surface limit efficiency is only $\eta_{\rm Th} = (1+3\bar{w}/2)/(1+\bar{w})^2$. The discrepancy led Theodorsen to state in the Preface of Ref. 1, and again in Chapter 4 in the paragraph on the partition of energy losses, that "the actuator disk does not constitute a true limiting case of the propeller." In the present slipstream model with rolled-up vortex sheets and swirl, this flaw is eliminated, justifying the statement that the actuator disk is a true limiting case of the propeller.

Effects of Viscosity

It is not clear beforehand whether a theoretical estimate of the efficiency (19) is an underestimation or overestimation. The effect of profile drag on the propeller blades surely increases the required power and decreases the efficiency. There is, however, an opposing effect: the effective redistribution of momentum by entrainment effects at the boundary of the slipstream may increase the efficiency, as suggested by Sparenberg[6] and van Holten[7]. The uncertainty about the effect leaves room for interpretation of any difference between prediction and experiment. Considering the example of the eight-bladed propfan, the rigid-sheet models yield a 3–3.5% overestimation, and the present rolled-up sheet model with swirl yields a 1.5% underestimation.

Conclusions

The present theory incorporating rollup and swirl in the slipstream is based on a realistic equilibrium model. It does not suffer from high pressure in the slipstream. In the actuator limit, $J = 0$, its values of thrust and power coefficients and of the efficiency approach the established values of actuator disk theory.

The actuator disk must be considered as a true limiting case of the propeller.

In the example of Hanson's eight-bladed propfan, the rigid-sheet methods yielded values for the efficiency that are 3–3.5% above the experimental value; the present theory yields an efficiency that is only 1.5% below the measured value. This result is interpreted as a support for the amended theory incorporating rollup and swirl.

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Equivalence Between Sideslip and Roll in Wind-Tunnel Model Testing

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Introduction

ASYMMETRIC orientations of fighter aircraft with respect to the direction of motion occur frequently during maneuvering; similar situations may arise during aerobatic performances, as well as when aircraft are subjected to strong crosswinds. The aerodynamic and aeroelastic performance of an aircraft and its components depends strongly on its orientation, and substantial tests have been performed to identify, in particular, the effects of sideslip. The effect of steady roll, which is equally important, has not been as well documented, although the rolling stability of aircraft has been a main issue of concern. Most of the quantitative information on aircraft aerodynamics has been collected through wind-tunnel testing, in which an aircraft model is mounted on a sting that can be rotated by one or more angles. It is obvious from economic and time considerations that the design of the mounting system should be as simple as possible and that the number of necessary tests should be minimal. Therefore, any procedure that may facilitate the design of model mounting systems or the extraction of aerodynamic information at one aircraft orientation from that at a more convenient one would be highly desirable. Although it is well known[1] that any orientation of an aircraft with respect to the direction of the wind (freestream) can be uniquely specified by the values of two angles, the angle of attack and the sideslip angle, and although a general compilation of aerodynamic axes systems and geometric relationships permitting the transfer from one system to another is available,[2] the equivalence between orientations obtained by a combination of pitching–yawing and by a combination of pitching–rolling has not been specifically addressed in the available literature. The purpose of this Note is to derive such a relationship and to clarify its usefulness in wind-tunnel testing.

Derivation of Equivalent Angle Relationships

Consider the aircraft shown schematically in Fig. 1. The body axes, x, y, and z, are Cartesian axes defined such that x is the longitudinal axis of the aircraft, y is the lateral axis, usually in the plane of the wings, and z is in the plane of symmetry of the aircraft and perpendicular to the other two. Let $\boldsymbol{V}$ be the velocity vector of the freestream and u, v, and w be its projections on the x, y, and z axes, respectively. The angle of attack α is defined as the angle between the projection of the wind axis on the x–z plane and the x axis, such that

$$a = \tan^{-1}(w/u) \tag{1}$$

The sideslip angle is defined as the angle between the wind axis and its projection on the x–z plane, such that

$$\beta = \sin^{-1}(v/V) \tag{2}$$

Then, the total or complex angle of attack α^* is defined as the angle between the wind axis and the x axis, such that

$$a^* = \cos^{-1}(u/V) \tag{3}$$

Let x_0, y_0, and z_0 (Fig. 1) be the directions of the body axes that correspond to an orientation of the aircraft such that the aircraft axis x_0 is aligned with the wind direction. Any arbitrary orientation of the aircraft with respect to the wind direction may be achieved by first rotating the aircraft about the y_0 axis, by an angle of attack α, and then about the z_0 axis, by a sideslip angle β. The same relative orientation may be achieved by first rotating the aircraft about the y_0 axis by an angle of attack α^*, and then about the x axis by a roll angle φ, which is, thus, defined as the angle between the y axis and the y_0 axis. These two orientations are equivalent because the corresponding body axes, although not parallel to each other, form the same angles with the vector $\boldsymbol{V}$ and, therefore, result in equal corresponding components of the wind vector on the body axes. The two sets of body axes can be made parallel by a rigid-body rotation of the latter system (namely, the one obtained by a set of α^*–φ rotations) about the freestream direction by an angle φ.

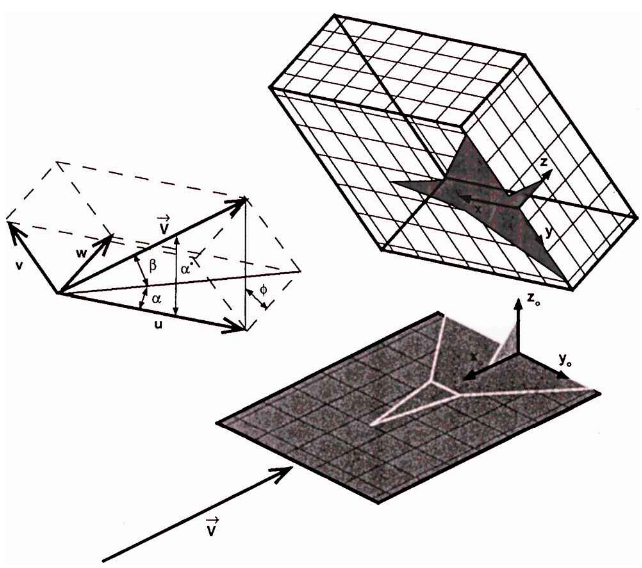

Fig. 1 Two aircraft orientations and the components of the freestream velocity vector (wind) with respect to the body-fixed coordinate axes.

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